STUDYING DISORDERED DYNAMICAL SYSTEMS USING COMBINATORIAL AND ALGEBRAIC TOPOLOGY

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Abstract

This dissertation investigates various dynamical systems through the frameworks of combinatorial and algebraic topology and geometry, with the goal of understanding how physical dynamical processes imprint themselves on the topology of the system. By developing a structured approach for analyzing topological information, this research applies these methods to both simulated and empirical dynamical data. The study encompasses a wide range of physical processes, beginning with an exploration of crack mosaics formed by drying solutions/suspensions of different materials. These are characterized by relatively simple patterns, and the study progressively extends to more intricate problems, such as flow patterns in fluid dynamical systems.

A key contribution of this work is the construction of topological and geometric tools that capture the essential features of dynamical systems, enabling their classification based on topological and geometric similarities and differences. For crack mosaics, a four-parameter tuple space, denoted as (n, v, D, λ), was introduced. This combinatorial and geometric framework successfully investigates and distinguishes crack pattern characteristics according to material properties of the drying system. In the analysis of spatio-temporal evolution in fluid dynamics, which is more involved, a novel topological tool, termed the Euler Characteristic Surface (ECS), was developed. The ECS encapsulates the topological signature of a dynamical system and provides a unique descriptor for it.

To quantify the similarity and dissimilarity between ECS representations, a new metric referred to as the "Euler Metric" was introduced, facilitating the comparison of dynamical systems through their topological characteristics. The effectiveness of these topological constructs has been thoroughly analyzed, with results that validate their ability to capture and distinguish dynamical behavior.

The dissertation concludes by establishing the mathematical stability of the proposed topological constructs and by drawing connections to widely-used techniques in Topological Data Analysis (TDA).